

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

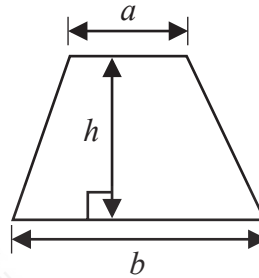
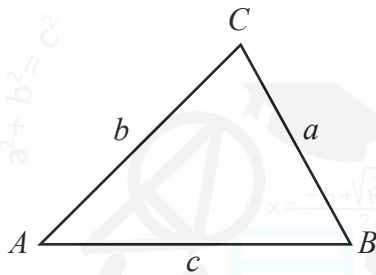
Sum to n terms, $S_n = \frac{n}{2} [2a + (n - 1)d]$

The quadratic equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium = $\frac{1}{2}(a + b)h$

**Trigonometry**

In any triangle ABC

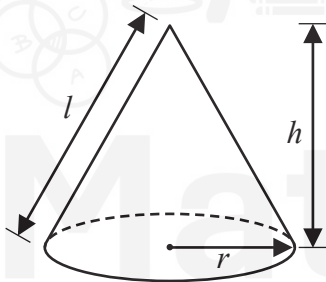
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$

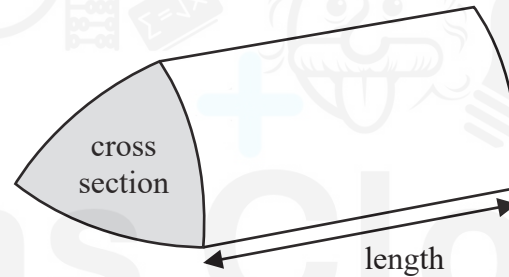
Volume of cone = $\frac{1}{3} \pi r^2 h$

Curved surface area of cone = $\pi r l$



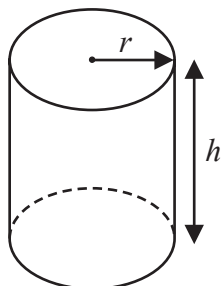
Volume of prism

= area of cross section \times length



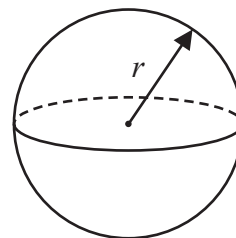
Volume of cylinder = $\pi r^2 h$

Curved surface area of cylinder = $2\pi r h$



Volume of sphere = $\frac{4}{3} \pi r^3$

Surface area of sphere = $4\pi r^2$



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Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Here are six cards.

Five of the cards have a number written on them.

16	15	3	2	9	
----	----	---	---	---	--

Work out the number that should be written on the last card so that the mean of the six numbers will be 11

Add then \div for the mean

$$\frac{16 + 15 + 3 + 2 + 9 + \text{\textit{§inal card value}}}{6} = 11$$

$$\frac{45 + \text{\textit{§inal card value}}}{6} = 11$$

$$45 + \text{\textit{§inal card value}} = 66$$

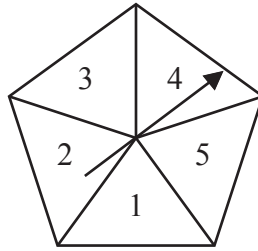
$$\text{\textit{§inal card value}} = 21$$

21

(Total for Question 1 is 3 marks)



2 Here is a biased spinner.



The table gives information about the probability that, when the spinner is spun once, it will land on each number.

Number	1	2	3	4	5
Probability	$2x$	0.27	0.04	x	0.12

Alexis is going to spin the spinner 400 times.

Work out an estimate for the number of times the spinner will land on an odd number.

first, work out the probabilities for each number:

$$\text{Total probability} = 1$$

$$1 - 0.27 - 0.04 - 0.12$$

$$= 0.57$$

$$2x + x = 0.57$$

$$3x = 0.57$$

$$x = 0.19$$

$$\text{Probability of landing on 1} : 2 \times 0.19 = 0.38$$

$$\text{Probability of landing on 4} : 0.19$$

$$\begin{aligned} \text{Sum of the odd} \\ \text{number probabilities} : 0.38 + 0.04 + 0.12 \\ = 0.54 \end{aligned}$$

$$400 \times 0.54 = 216$$

.....216.....

(Total for Question 2 is 4 marks)

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- 3 Norberto sells white loaves of bread and brown loaves of bread.

He sells a total of 200 loaves such that

the number of white loaves sold : the number of brown loaves sold = 3 : 2

Norberto sells the white loaves for £1.50 each.

He sells the brown loaves for £1.75 each.

40% of the price of a white loaf is profit.

60% of the price of a brown loaf is profit.

Work out Norberto's total profit when he sells all 200 loaves.

first, calculate number of each type:

$$(200) + (3+2) = 40$$

$$3 : 2$$

$$40 \times 3 : 40 \times 2$$

$$120 : 80$$

120 white loaves

80 brown loaves

Total sales:

$$120 \times 1.50 = £180$$

$$80 \times 1.75 = £140$$

Profit:

$$(180 \times 0.4) + (140 \times 0.6)$$

$$= 72 + 84$$

$$= £156$$

£ 156

(Total for Question 3 is 5 marks)



4 Show that $2\frac{1}{3} \div 5\frac{1}{4} = \frac{4}{9}$

Converting fractions:

$$2 = \frac{3}{3} \times 2 = \frac{6}{3} + \frac{1}{3} = \frac{7}{3}$$

$$5 = \frac{4}{4} \times 5 = \frac{20}{4} + \frac{1}{4} = \frac{21}{4}$$

becomes:

$$\frac{7}{3} \div \frac{21}{4}$$

Keep $7/3$

Flip $21/4$ to $4/21$

change \div to \times :

$$\frac{7}{3} \times \frac{4}{21} = \frac{4}{9} \text{ as required}$$

(Total for Question 4 is 3 marks)

- 5 Slavomir invests 5200 euros in a savings account for 4 years. He gets 2.5% per year compound interest.

Work out how much money Slavomir will have in the savings account at the end of 4 years.

Give your answer correct to the nearest euro.

Compound interest formula:

$$\text{Initial amount} \times \left(\frac{\text{interest rate}}{100} + 1 \right)^{\text{number of years}}$$

$$5200 \times (1.025)^4$$

$$= 5739.83$$

To the nearest euro:

5740 euros

..... 5740 euros

(Total for Question 5 is 3 marks)



- 6 The diagram shows a solid wooden cylinder.

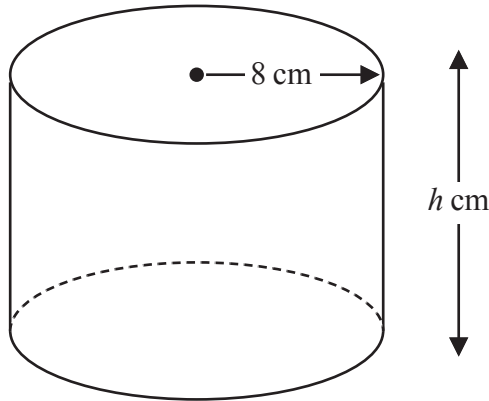


Diagram NOT
accurately drawn

The cylinder has radius 8 cm and height h cm.
The volume of the cylinder is 1208 cm^3

- (a) Work out the value of h .
Give your answer correct to the nearest whole number.

$$\text{Volume of cylinder} = \pi \times r^2 \times h$$

$$1208 = \pi \times 8^2 \times h$$

$$h = 1208 \div \pi \times 8^2$$

$$h = 6 \text{ To nearest whole number}$$

$$h = 6 \dots\dots\dots (2)$$

The density of the wood is 1.25 g/cm^3

- (b) Work out the mass of the cylinder.
Give your answer in kilograms.

$$\text{Mass} = \text{Volume} \times \text{density}$$

$$= 1208 \times 1.25$$

$$= 1510 \text{ g}$$

$$\div 1000 \text{ for kg}$$

$$= 1.51 \text{ kg}$$

$$\dots\dots\dots 1.51 \dots\dots\dots \text{ kilograms} \quad (2)$$

(Total for Question 6 is 4 marks)

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7 (a) Simplify $g^9 \div g^2$

When \div , we - powers
 $= g^{9-2} = g^7$

$$g^7$$

(1)

(b) Expand $5k^2(k^3 + 4)$

Multiply powers when expanding brackets:
 $5k^2 \times k^3 + 20k^2$
 $= 5k^6 + 20k^2$

$$5k^6 + 20k^2$$

(2)

(c) (i) Factorise $x^2 - 2x - 63$

What + to get -2
 and x to get -63?

consider +7, -9:

$$7 \times (-9) = -63$$

$$7 - 9 = -2$$

factorised: $(x+7)(x-9)$

$$(x+7)(x-9)$$

(2)

(ii) Hence, solve $x^2 - 2x - 63 = 0$

$$(x+7)(x-9) = 0$$

$$\text{so } x+7=0 \text{ or}$$

$$x-9=0$$

$$\text{so } x = -7 \text{ or}$$

$$x = 9$$

$$x = -7, 9$$

(1)

(d) Solve the inequality $7 - 2y < 3y - 12$

$$\begin{aligned} 7 - 2y &< 3y - 12 && \left. \begin{array}{l} \\ \end{array} \right\} +12 \\ 19 - 2y &< 3y && \\ 19 &< 5y && \left. \begin{array}{l} \\ \end{array} \right\} +2y \\ \frac{19}{5} &< y && \left. \begin{array}{l} \\ \end{array} \right\} \div 5 \end{aligned}$$

$$y > \frac{19}{5}$$

(3)

(Total for Question 7 is 9 marks)



- 8 The diagram shows a trapezium, $ABCD$

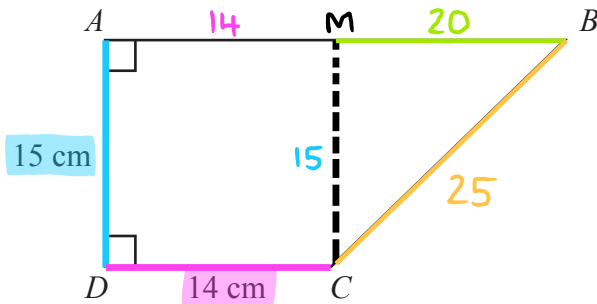


Diagram NOT
accurately drawn

DAB and ADC are right angles.

$$AD = 15 \text{ cm}$$

$$DC = 14 \text{ cm}$$

The area of the trapezium is 360 cm^2

Work out the perimeter of the trapezium.

$$\begin{aligned} \text{Area of Square} &: 15 \times 14 \\ &= 210 \end{aligned}$$

$$\begin{aligned} \text{area of triangle BMC} &: 360 - 210 \\ &= 150 \end{aligned}$$

$$\text{area triangle} = \frac{CM \times BM}{2}$$

$$\frac{15 \times BM}{2} = 150$$

$$15BM = 300$$

$$BM = 20$$

find BC using pythagoras:

$$a^2 + b^2 = c^2$$

$$20^2 + 15^2 = BC^2$$

$$BC = \sqrt{20^2 + 15^2}$$

$$BC = 25$$

Perimeter:

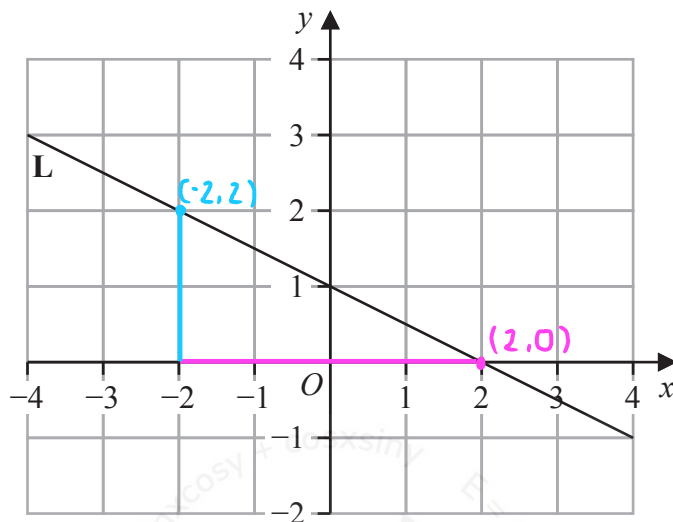
$$\begin{aligned} &15 + 14 + 14 + 20 + 25 \\ &= 88 \text{ cm} \end{aligned}$$

88 cm

(Total for Question 8 is 6 marks)



- 9 Line L is drawn on the grid.



Find an equation for L

Give your answer in the form $y = mx + c$

To find m : $\frac{y_2 - y_1}{x_2 - x_1}$

Pick 2 points on line:

$(-2, 2)$ $(2, 0)$
 x_1, y_1 x_2, y_2

$$= \frac{0 - 2}{2 - (-2)} = -\frac{1}{2}$$

so $y = -\frac{1}{2}x + c$

for $(-2, 2)$

$$2 = -\frac{1}{2}(-2) + c$$

$$2 = 1 + c$$

$$c = 1$$

So we have: $y = -\frac{1}{2}x + 1$

$$y = -\frac{1}{2}x + 1$$

(Total for Question 9 is 3 marks)

- 10 Here are the numbers of goals scored by a hockey team in its 11 games this season.

0 1 2 2 3 4 4 6 7 9 11

Work out the interquartile range of the numbers of goals.

LQ = bottom 25%,

$$= 11 \times 0.25 = 2.75 \approx 3^{\text{rd}} \text{ value}$$

UQ = Top 25%,

$$= 11 \times 0.75 = 8.25 \approx 9^{\text{th}} \text{ value}$$

5

$$7 - 2 = 5$$

(Total for Question 10 is 2 marks)



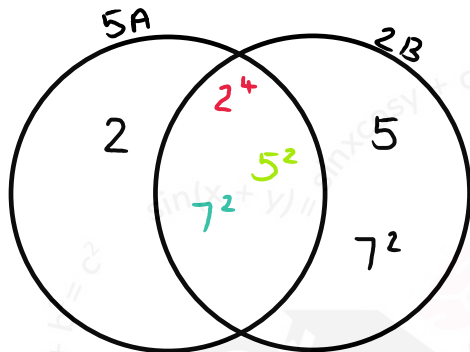
$$11 \quad A = 2^5 \times 5 \times 7^2 \quad \left. \begin{array}{l} 5A = 2^5 \times 5^2 \times 7^2 \\ B = 2^3 \times 5^3 \times 7^4 \\ 2B = 2^4 \times 5^3 \times 7^4 \end{array} \right\} \text{First account for additional terms from multiplication}$$

- (a) Write down the highest common factor (HCF) of $5A$ and $2B$
Give your answer as a product of prime factors.

Use Venn diagram technique:

$$5A = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7 \times 7$$

$$2B = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7 \times 7$$



$$= 2^4 \times 5^2 \times 7^2$$

$$= 2^4 \times 5^2 \times 7^2 \quad (2)$$

$$A = 2^5 \times 5 \times 7^2$$

$$B = 2^3 \times 5^3 \times 7^4$$

- (b) Work out the value of $(AB)^2$
Give your answer as a product of prime factors.

$A \times B$ ← add the powers of the same base. e.g. $2^2 \times 2^3 = 2^5$

$$AB = (2^5 \times 5 \times 7^2) \times (2^3 \times 5^3 \times 7^4)$$

$$= 2^{5+3} \times 5^{1+3} \times 7^{2+4}$$

$$= 2^8 \times 5^4 \times 7^6$$

for $2AB$, multiply each power by 2.

$$= 2^{16} \times 5^8 \times 7^{12}$$

$$2^{16} \times 5^8 \times 7^{12} \quad (2)$$

(Total for Question 11 is 4 marks)



12 Solve the simultaneous equations

$$\begin{aligned}4x + 3y &= 9.6 \\6x + 5y &= 16.8\end{aligned}$$

Show clear algebraic working.

(1) $4x + 3y = 9.6$ ($\times 3$)

(2) $6x + 5y = 16.8$ ($\times 2$)

$$12x + 9y = 28.8 \quad -$$

$$12x + 10y = 33.6$$

$$-y = -4.8$$

$$y = 4.8$$

plug $y = 4.8$ into (1)
to solve for x :

$$4x + 3(4.8) = 9.6$$

$$4x + 14.4 = 9.6$$

$$4x = -4.8$$

$$x = -1.2$$

Same sign so we subtract the equations. We want to make either the x or y values the same, to then be left with one to solve for. Let's make the x values match:

$$x = -1.2$$

$$y = 4.8$$

(Total for Question 12 is 4 marks)

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13

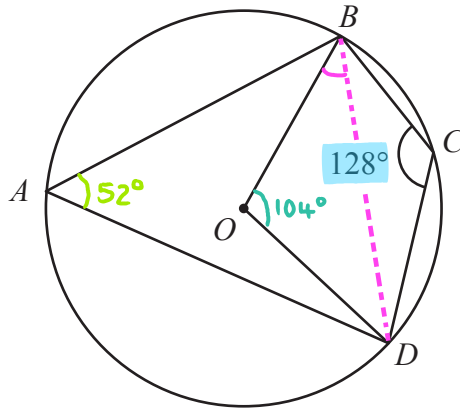


Diagram **NOT**
accurately drawn

A, B, C and D are points on a circle, centre O

Angle $BCD = 128^\circ$

Work out the size of angle OBD

Give a reason for each stage of your working.

$$BAD = 180 - 128 = 52^\circ$$

Opposite angles of cyclic quadrilateral sum to 180°

$$BOD = 2 \times 52 = 104^\circ$$

Angle at centre is $\times 2$ angle at circumference

$$OBD = \frac{180 - 104}{2}$$

$$= 38^\circ$$

Angles at base of isosceles triangle are equal.

$$\text{angle } OBD = 38^\circ$$

(Total for Question 13 is 5 marks)



14 (a) Expand and simplify $(3x+1)(2-x)(4+x)$

$$\begin{array}{r|l} 3x & 1 \\ \hline 2 & 6x \quad 2 \\ -x & -3x^2 \quad -2x \end{array} = 3x^2 + 4x + 2$$

$$\begin{array}{r|l} 3x^2 & 4x & 2 \\ \hline 4 & 12x^2 & 16x & 8 \\ x & 3x^3 & 4x^2 & 2x \end{array} = -3x^3 - 7x + 22x + 8$$

$$\frac{-3x^3 - 7x + 22x + 8}{(3)}$$

(b) Simplify fully $\left(\frac{a^3b}{a^9b^5}\right)^{\frac{1}{2}}$

Remove the - sign by flipping the fraction.

$$= \left(\frac{a^9b^5}{a^3b}\right)^{\frac{1}{2}}$$

Simplify by - powers

$$= (a^6b^4)^{\frac{1}{2}}$$

÷ each power by 2

$$= a^3b^2$$

$$\frac{a^3b^2}{(3)}$$

(Total for Question 14 is 6 marks)

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15 The diagram shows isosceles triangle EFG

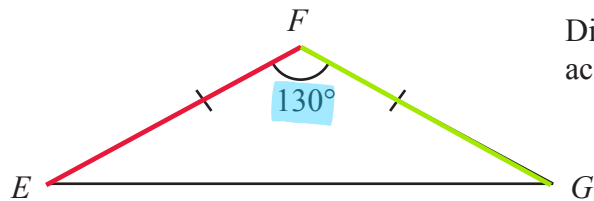


Diagram NOT
accurately drawn

$$EF = GF$$

$$\text{Angle } EFG = 130^\circ$$

The area of triangle EFG is 74 cm^2

Work out the length of EF

Give your answer correct to 3 significant figures.

$$\text{Area: } \frac{1}{2} EF \sin EFG$$

$$74 = \frac{1}{2} EF \times \sin 130$$

$$\sqrt{\frac{2 \times 74}{\sin 130}} = EF$$

$$EF = 13.9 \text{ cm}$$

To 3 significant figures

13.9 cm

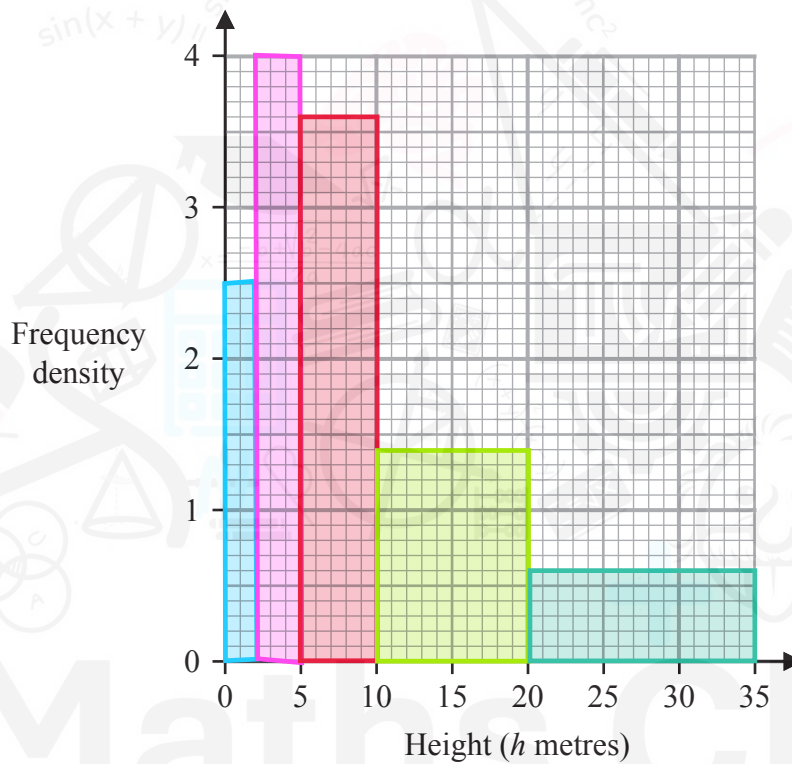
(Total for Question 15 is 3 marks)



16 The table gives information about the heights, in metres, of the trees in a park.

Height (h metres)	Frequency	Class width	Frequency density
$0 < h \leq 2$	5	$2 - 0 = 2$	$5 \div 2 = 2.5$
$2 < h \leq 5$	12	$5 - 2 = 3$	$12 \div 3 = 4$
$5 < h \leq 10$	18	$10 - 5 = 5$	$18 \div 5 = 3.6$
$10 < h \leq 20$	14	$20 - 10 = 10$	$14 \div 10 = 1.4$
$20 < h \leq 35$	9	$35 - 20 = 15$	$9 \div 15 = 0.6$

On the grid, draw a histogram for this information.



(Total for Question 16 is 3 marks)

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$$17 \text{ (a) } (\sqrt[4]{k^{12}})^5 = k^n$$

Find the value of n

$$= 12 \div 4 = 3$$

$$= (k^3)^5$$

$$= 3 \times 5$$

$$= k^{15}$$

$$\text{So } n = 15$$

$$n = 15 \dots\dots\dots$$

(1)

(b) Express $\frac{7}{2-\sqrt{3}}$ in the form $\sqrt{c} + d$ where c and d are integers.

Show your working clearly.

rationalise the denominator:

$$\frac{7}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$\begin{array}{r|l} 2 & 2+\sqrt{3} \\ 2 & 4 \quad 2\sqrt{3} \\ -\sqrt{3} & 2\sqrt{3} \quad -3 \end{array} = 1$$

$$= \frac{14 + 7\sqrt{3}}{1}$$

$$= \sqrt{147} + 14 \text{ as required}$$

$$\dots\dots\dots \sqrt{147} + 14 \dots\dots\dots$$

(3)

(Total for Question 17 is 4 marks)

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18 The diagram shows two similar vases, A and B

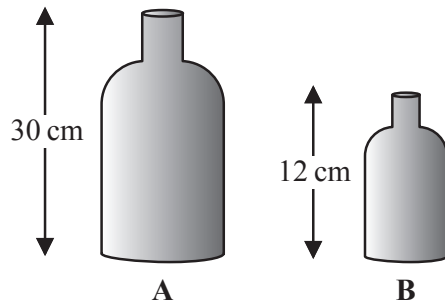


Diagram NOT
accurately drawn

The height of vase A is 30 cm

The height of vase B is 12 cm

Given that

$$\text{surface area of vase A} - \text{surface area of vase B} = 178.5 \text{ cm}^2$$

find the surface area of vase A

ratio of heights:

$$30 : 12$$

$$5 : 2$$

$$2 \div 5 = 0.4$$

so length scale factor = 0.4

$$\text{so SA scale factor} = 0.4^2 = \frac{4}{25}$$

$$\text{so } \frac{4}{25} A = B$$

$$A - \frac{4}{25} A = 178.5$$

$$\frac{21}{25} A = 178.5$$

$$A = 212.5$$

$$\dots\dots\dots 212.5 \dots\dots\dots \text{cm}^2$$

(Total for Question 18 is 4 marks)

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19 A curve C has equation $y = x^3 - 8x^2 - 12x + 5$

Curve C has exactly two stationary points, one at point A and one at point B such that

x coordinate of point $A > x$ coordinate of point B

Find the coordinates of point A

Show clear algebraic working.

First, differentiate, "x power, -1 power"

$$y = x^3 - 8x^2 - 12x + 5$$

$$\frac{dy}{dx} = 3x^2 - 16x - 12 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{+16 \pm \sqrt{16^2 - 4(3)(-12)}}{2(3)}$$

$$x = 6, x = -\frac{2}{3}$$

given: A x coord $>$ B x coord

Take $x = 6$

plug back into $y = x^3 - 8x^2 - 12x + 5$

$$y = 6^3 - 8(6)^2 - 12(6) + 5$$

$$y = -139$$

$$= (6, -139)$$

(6 , -139)

(Total for Question 19 is 5 marks)



- 20 (a) Express $2x^2 - 11x + 9$ in the form $a(x - b)^2 - c$ where a , b and c are numbers to be found.

We complete the square:

factor out 2:

$$2(x^2 - \frac{11}{2}x) + 9$$

complete the square $\left. \begin{array}{l} \frac{11}{2} \div 2 \\ -(\frac{11}{2})^2 \end{array} \right\}$

$$2\left[\left(x - \frac{11}{4}\right)^2 - \frac{121}{16}\right] + 9$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + 9 \quad \left. \begin{array}{l} \\ \end{array} \right\} \times 2$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$$

(3)

minimum point: $\left(\frac{11}{4}, -\frac{49}{8}\right)$

The curve C has equation $y = 2(x - 3)^2 - 11(x - 3) + 9$

The point P is the minimum point on C

- (b) Find the coordinates of P

$$f(x) = 2x^2 - 11x + 9$$

$$f(x-3) = 2(x-3)^2 - 11(x-3) + 9$$

Translation of f +3 x value

$$\left(\frac{11}{4} + 3, -\frac{49}{8}\right)$$

$$= \left(\frac{23}{4}, -\frac{49}{8}\right)$$

$$\left(\frac{23}{4}, -\frac{49}{8}\right)$$

(2)

(Total for Question 20 is 5 marks)



21 There are 25 counters in a bag such that

6 counters are blue

x counters are orange, where $x > 9$

the rest of the counters are pink

Maalam takes at random two of the counters from the bag.

The probability that Maalam takes one orange counter and one pink counter is $\frac{22}{75}$

Calculate the probability that Maalam takes 2 pink counters from the bag.
Show clear algebraic working.

$$\frac{x}{25} \times \frac{25 - (x + 6)}{24} = \frac{22}{75} \quad \left. \begin{array}{l} \times 25 \\ \times 24 \end{array} \right\}$$

$$2 \times \left(\frac{x \times 25(25 - (x + 6))}{24} \right) = \frac{22}{75} \times 25 \quad \left. \begin{array}{l} \times 24 \\ \times 24 \end{array} \right\}$$

$$2 \times (24x \times 25(25 - (x + 6))) = \frac{22}{75} \times 25 \times 24$$

Simplifying gives:

$$2x^2 - 38x + 176 = 0$$

$$x^2 - 19x + 88 = 0$$

$$(x - 11)(x - 8) = 0$$

$$x = 11 \text{ or } x = 8$$

given $x > 9$ take $x = 11$

and number
pink
counters = 8

$$P(\text{picks 2 pink}) = \frac{8}{25} \times \frac{7}{24} = \frac{7}{75}$$

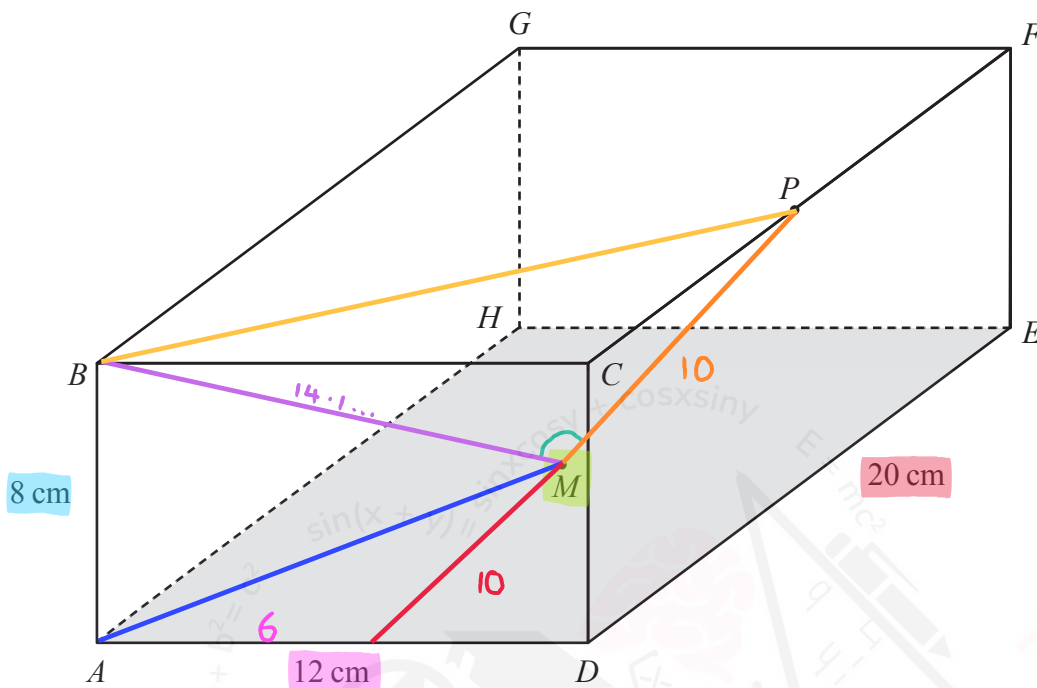
7175

(Total for Question 21 is 5 marks)



22 The diagram shows a cuboid $ABCDEFGH$ with horizontal base $ADEH$

Diagram **NOT** accurately drawn



$$AB = 8 \text{ cm} \quad AD = 12 \text{ cm} \quad DE = 20 \text{ cm}$$

M is the midpoint of the base $ADEH$ and P is the midpoint of the edge CF

Work out the size of angle BMP

Give your answer correct to one decimal place.

Using Pythagoras:

$$MP = \sqrt{8^2 + 6^2} = 10$$

$$BM = \sqrt{AM^2 + 6^2}$$

$$= \sqrt{(\sqrt{10^2 + 6^2})^2 + 8^2}$$

$$= 14.1 \dots$$

$$BP = \sqrt{12^2 + 10^2}$$

$$= 15.6 \dots$$

plug into cosine rule:

$$\text{Cosine Rule } a^2 = b^2 + c^2 - 2bc \cos A \quad \leftarrow \text{BMP}$$

$$(15.6)^2 = 10^2 + 14.1^2 - 2(10)(14.1) \cos \text{BMP}$$

$$\cos^{-1} \left(\frac{10^2 + 14.1^2 - 15.6^2}{2(10)(14.1)} \right) = \text{BMP}$$

$$= 78.6^\circ$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



23 Here are the first three terms of an arithmetic sequence.

$$(4x-14), (x+2), (7x-9)$$

Find, as an integer, the sum of the first 40 terms of the sequence.
Show clear algebraic working.

find x :

$$(7x-9) - (x+2) = (x+2) - (4x-14)$$

$$6x - 11 = -3x + 16$$

$$9x = 27$$

$$x = 3$$

Series becomes:

$$-2, 5, 12$$

$$d = 5 - (-2) = 7$$

$$a = -2$$

$$S_{40} = \frac{40}{2} [2a + (40-1)d]$$

$$= \frac{40}{2} [2(-2) + 39(7)]$$

$$= 5380$$

5380

(Total for Question 23 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS

